Using S.I. units

Specification references

- *2.1.2 a) b) c) d)*
- *M0.1 Recognise and make use of appropriate units in calculations*

Learning outcomes

After completing the worksheet you should be able to:

- show knowledge and understanding of base and derived S.I. units
- use equations to work out derived units
- use base units to check homogeneity of equations.

Introduction

Base quantities are measured in base units. These are units that are not based on other units. For example, mass is measured in kilograms and length is measured in metres. Other quantities have units which are derived from the base quantities. For example, the unit of density ($kg\,m^{-3}$) is derived from the kilogram and the metre.

The first example shows you how to use an equation to work out the unit of a derived quantity. The second example shows you how to check that an equation is homogeneous or, in other words, that its units are balanced.

Worked example

Question

What is the S.I. unit of speed?

Answer

Step 1

Identify the equation to use.

Speed is defined as: $\dfrac{\text{distance travelled}}{\text{time taken}}$

Step 2

Write the equation in terms of units.

The S.I. unit of speed is defined as: $\dfrac{\text{unit of distance travelled}}{\text{unit of time taken}}$

Step 3

Select the appropriate S.I. base units.

S.I. unit of distance = metre (m)

S.I. unit of time = second (s)

Step 4

Insert the S.I. base units into the equation.

$$\text{S.I. unit of speed} = \frac{\text{metre (m)}}{\text{time taken (s)}} = \text{metre per second} = \text{m s}^{-1}$$

Question

1 Work out the missing units, unit symbols and names, equations, and quantities in this table. (*1 mark* for each correct answer)

Physical quantity	Equation used	Unit	Derived unit symbol and name
frequency	$\frac{1}{\text{time period}}$	**a**	Hz hertz
volume	length^3	**b**	–
acceleration	$\frac{\text{velocity}}{\text{time}}$	**c**	–
force	mass × acceleration	kg m s^{-2}	**d**
work and energy	force × distance	**e**	J joule
voltage	$\frac{\text{energy}}{\text{electric charge}}$	J C^{-1}	**f**
electrical resistance	**g**	V A^{-1}	**h**
momentum	mass × velocity	**i**	**–**
impulse	force × time	**j**	**–**
k	$\frac{\text{force}}{\text{area}}$	**l**	Pa pascal
m	**n**	kg m^{-3}	**–**

Worked example

Question

Check that the equation: kinetic energy = $\frac{1}{2}mv^2$ is homogeneous.

Answer

Make sure you always state which side of the equation you are working on, left-hand side (LHS) or right-hand side (RHS).

Step 1

Start with the LHS. The unit of kinetic energy is the joule. Change this to base units.

LHS: $\mathrm{J = N\,m = kg\,m\,s^{-2} \times m = kg\,m^2\,s^{-2}}$

Step 2

Repeat Step 1 for the RHS.

RHS: units of $\frac{1}{2}mv^2$ are $\mathrm{kg \times (m\,s^{-1})^2 = kg\,m^2\,s^{-2}}$

(The constant, $\frac{1}{2}$, is a number with no units.)

Step 3

Don't forget to write your conclusion.

LHS = RHS so the equation is homogeneous.

We can't tell that there is a $\frac{1}{2}$ in the equation, so we cannot say that the equation is correct, only that it is homogeneous.

Questions

2 Use base units to show the equation $Q = It$ for electric charge passing a point in time t, when the electric current is I, is homogeneous. (*1 mark*)

3 Use base units to show that the equation $P = IV$ is homogeneous, where I is electric current, V is voltage, and P is power measured in watts (W).
(Hint: $\mathrm{1\,W = 1\,J\,s^{-1}}$) (*2 marks*)

4 The Earth's gravitational field strength, $g = 9.81\,\mathrm{N\,kg^{-1}}$, is also sometimes given as the acceleration due to gravity, $g = 9.81\,\mathrm{m\,s^{-2}}$. Show that these units are equivalent. (*1 mark*)

Maths skills links to other areas

You may also need to check equations are homogeneous wherever they are used in the specification – examples can be found in Chapter 3 *Motion*, and Topic 4.8 *Density and pressure*.

You can also use this method to help you decide whether you have remembered an equation correctly.

Homogeneity of physical equations

Specification references

- *2.1.2 a) b) d)*
- *M0.1, M2.2*

Introduction

In physics, all physical quantities are defined by mathematical relationships, and have both magnitude (size) and units. By analysing the mathematical relationships and the units of the quantities involved, we can determine if an equation could be correct. This is because the quantities on both sides of any valid equation must have the same overall units.

Learning outcomes

After completing the worksheet you should be able to:

- rearrange a range of equations to change the subject
- analyse equations to find the units for unknown values (constants or variables)
- compare both sides of an equation to determine if the units are homogeneous.

Background

The Système Internationale (abbreviated to S.I.) has been adopted as a consistent and well-defined system of measurement of physical quantities. It is based on a set of seven base units from which all other physical quantities can be defined. These S.I. base units are described in your student book, Topic 2.1 *Quantities and units*, and are summarised in Table 1.

Table 1 *S.I. base units*

Quantity	Base unit	Unit symbol
length	metre	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

The units of all other physical quantities can be expressed in terms of these base units. These derived units are defined by a set of fundamental relationships. For example, the velocity of an object is defined as *the rate of change of displacement*.

Expressed as a physical equation, this is $v = \dfrac{\Delta s}{\Delta t}$.

By considering the units of displacement and time we can determine the units for velocity. The unit of displacement is the metre (m) and the unit of time is the second (s). The equation shows us that velocity is a displacement divided by a time, and therefore the unit of velocity must be metres per second (m/s = m s^{-1}).

Some derived units have been given special names; usually these honour scientists who have had a specific influence on the development of physics knowledge and understanding.

Homogeneity

The units on both sides of an equation must be equivalent otherwise the equation cannot be correct. If the units are fundamentally different then the quantities on the different sides of the equal sign cannot be equivalent. A velocity cannot be the same as an acceleration: they have different units.

This homogeneity is a consequence of the precise definitions of quantities in physics, which ensures a logical and mathematical consistency. Note that it is possible for the units to be homogenous when the relationship itself is not correct.

The principle of homogeneity allows us to:

- deduce the units of an unknown value (constant or variable) in an equation
- check that an equation shows a possible physical relationship.

Task 1: deriving units

As equations must be homogeneous, it is possible to determine the dimensions of any factor (constant or variable) in an equation if the dimensions of the other factors are known.

Example

When stretching a spring, a constant called the spring constant (symbol *k*) connects the force used to stretch the spring (*F*), and the extension of the spring (*x*).

The relationship is written as $F = kx$.

What are the units for the constant *k*?

5 Rearrange the equation to make *k* the subject.

$$k = \frac{F}{x}$$

6 Write the equation in terms of units.

$$\text{units for } k = \frac{\text{unit of force}}{\text{unit of length}} = \frac{\text{N}}{\text{m}}$$

7 See that the units for k are N m^{-1} (newtons per metre).

8 You can continue the process to find the units in terms of only the base units. To do this we need to find the base units for force. We can use the relationship $F = ma$. The unit for mass is kg and the unit for acceleration is m s^{-2}. This shows that force has the unit kg m s^{-2}.

$$\text{units of } k = \frac{\text{N}}{\text{m}} = \frac{\text{kg m s}^{-2}}{\text{m}} = \text{kg s}^{-2}$$

Questions

1 Determine the base units for these derived quantities. You must show your working.

- **a** Energy (joule) (*1 mark*)
- **b** Power (watt) (*1 mark*)
- **c** Potential difference (energy per unit charge) (*1 mark*)
- **d** Resistance (ohm) (*1 mark*)

2 The resistivity (ρ) of a material is defined by the equation

$$R = \rho\frac{l}{A}, \text{ or resistance} = \text{resistivity} \times \frac{\text{length}}{\text{cross-sectional area}}.$$

- **a** Rearrange the equation so that ρ is the subject. (*1 mark*)
- **b** Find the units of resistivity. Show all your working. (*1 mark*)
- **c** Find the units of resistivity in terms of the base units. Show all your working. (*1 mark*)

3 The force of gravitational attraction between two masses, m_1 and m_2, separated by a distance, r, can be determined by the equation $F = G\frac{m_1 m_2}{r^2}$, where G is a constant known as the gravitational constant.

- **a** Rearrange the equation so that G is the subject. (*1 mark*)
- **b** Find the units of G in terms of base units. (*1 mark*)

Task 2: analysing equations

By checking that the units of both sides of an equation are the same (homogeneous) we can test to see if an equation *may* be correct, or at least rule out incorrect equations.

Example

When a pendulum swings, the period of the pendulum swing (T) is related to the length of the pendulum (l) and the acceleration due to gravity by the following relationship.

$$T = 2\pi\sqrt{\frac{l}{g}}$$

To verify that the units are homogeneous, we look at the units for the length (m) and the acceleration due to gravity (m s^{-2}). The constant 2π is a pure number and so has no dimensions.

$$s = \sqrt{\frac{m}{m\,s^{-2}}} = \sqrt{\frac{1}{s^{-2}}} = \sqrt{\frac{s^2}{1}} = s$$

The analysis shows that the units on both sides of the equals sign are the same, and so the equation may be valid.

As mentioned earlier, this analysis does not prove that the equation is correct; only that it could be correct. An analysis of $T = 4\pi\sqrt{\frac{l}{g}}$ would show that the dimensions are homogeneous but the relationship is not correct.

Questions

1 Confirm whether the following equations have homogeneous units. You must show your working in each case.

a For changes in gravitational potential energy: $\Delta E_P = mg\Delta h$ (*2 marks*)

b For kinetic energy: $E_K = \frac{1}{2}mv^2$ (*2 marks*)

c In electrical circuits: $P = I^2R$ (*2 marks*)

d The time period, T, for a mass, m, oscillating on a spring with a spring constant k: $T = 2\pi\sqrt{\frac{k}{m}}$ (*2 marks*)

Determining uncertainty

Specification references

- *2.2.1 c)*
- *M0.3 Use ratios, fractions, and percentages*
- *M1.5 Identify uncertainties in measurements and use simple techniques to determine uncertainty when data are combined by addition, subtraction, multiplication, division, and raising to powers*

Learning outcomes

After completing the worksheet you should be able to:

- demonstrate knowledge and understanding of percentage errors and uncertainties
- evaluate absolute and percentage uncertainties
- determine uncertainty when data are combined by addition, subtraction, multiplication, division, and raising to powers.

Percentage uncertainties

Introduction

When something is measured there will always be a small difference between the measured value and the true value. There are several possible reasons for uncertainty in measurements, including the difficulty of taking the measurement, the precision of the measuring instrument (for example, due to the size of the scale divisions), and the natural variation of the quantity being measured. The word 'uncertainty' is generally used in preference to 'error', because 'error' implies something that is wrong – mistakes in making measurements should be avoided, and are not included in the uncertainty.

A measurement of 2.8 g on a scale with divisions of 0.1 g means the value is closer to 2.8 g than 2.7 g or 2.9 g. If the measurement were exactly half-way between 2.8 g and 2.9 g you would round up and record 2.9 g, so 2.8 g is anything from 2.75 g up to, but not including, 2.85 g, and the measurement is written 2.8 (± 0.05) g. The '(± 0.05) g' is called the absolute uncertainty.

The percentage uncertainty in a measured value is calculated as shown below.

$$\text{percentage uncertainty} = \frac{\text{uncertainty}}{\text{measured value}} \times 100\%$$

Worked example

Question

a The distance from **A** to **B** is carefully measured as 7.500 m, using a 10 m tape measure marked in millimetre increments.

  i Deduce the absolute uncertainty in the measurement.

  ii Determine the percentage uncertainty in the measurement.

b The distance from **B** to **C** is measured as 6.500 m using a stick 1 m in length with no scale divisions, in difficult conditions.

  i Deduce the absolute uncertainty in the measurement.

  ii Determine the percentage uncertainty in the measurement.

c Calculate the absolute uncertainty in the total distance from **A** to **B** to **C**.

d Calculate the percentage uncertainty in the total distance from **A** to **B** to **C**.

Answer

a i *Step 1*

Consider the start point (**A**) and end point (**B**) of the measurement, the scale division size, and the difficulty of measuring. There will be an uncertainty in the measurement both at the start point (**A**) and the end point (**B**).

The uncertainty in the measurement = 2 mm
(If you measure a shorter length with a 30 cm ruler, there would be an uncertainty of 0.5 mm at each end, resulting in a 1 mm uncertainty overall.)

Step 2

Write out the measurement with its absolute uncertainty. The uncertainty has the same unit as the measurement.

The distance **AB** is 7.500 (± 0.002) m.

ii *Step 3*

Calculate the percentage uncertainty using the equation:

$$\text{percentage uncertainty} = \frac{\text{uncertainty}}{\text{measured value}} \times 100\%$$

$$\text{percentage uncertainty} = \frac{0.002}{7.500} \times 100\% = 0.03\%$$

b i *Step 4*

Consider the start point (**B**) and end point (**C**) of the measurement, the scale division size, and the difficulty of measuring. There will be an uncertainty in the measurement at the start point (**B**) and at the end point (**C**). Because the metre stick has no scale divisions, you can only estimate to the nearest half a metre.

The uncertainty in the measurement = 0.5 m

Step 5

Write out the measurement with its absolute uncertainty.

The distance **BC** is 6.5 ($\pm$0.5) m.

Step 6

The percentage uncertainty $= \dfrac{0.5}{6.5} \times 100\%$

$= 7.69\%$

$= 8\%$ (to nearest %)

Unless the percentage uncertainty is less than 1%, it is acceptable to quote percentage uncertainties to the nearest whole number.

c *Step 7*

For the distance **ABC** the two measurements are added. The overall absolute uncertainty will be the sum of the individual absolute uncertainties.

uncertainty in **ABC** = uncertainty in **AB** + uncertainty in **BC**

$= 0.002\,\text{m} + 0.5\,\text{m}$

$= 0.5\,\text{m}$ (since 0.002 is insignificant compared to 0.5)

d *Step 8*

To find the percentage uncertainty, first calculate the measured value of **ABC**.

ABC $= 7.5 + 6.5 = 14.0\,\text{m}$

Step 9

Calculate the percentage uncertainty using the equation:

$$\text{percentage uncertainty} = \frac{\text{uncertainty}}{\text{calculated value}} \times 100\%$$

$$\text{percentage uncertainty} = \frac{0.5}{14.0} \times 100\%$$

$= 3.57\%$

$= 4\%$ (to nearest %)

Questions

2 Write down these measurements with their absolute uncertainty.

- **a** 6.0 cm length measured with a ruler marked in mm (*1 mark*)
- **b** 0.642 mm diameter measured with a digital micrometer (*1 mark*)
- **c** 36.9 °C temperature measured with a thermometer which has a quoted accuracy of: '± 0.1 °C (34 to 42 °C), rest of range ± 0.2 °C'. (*1 mark*)

3 Calculate the percentage uncertainty in these measurements.

- **a** 5.7 ± 0.1 cm (*1 mark*)
- **b** 2.0 ± 0.1 A (*1 mark*)
- **c** 450 ± 2 kg (*1 mark*)
- **d** 10.60 ± 0.05 s (*1 mark*)
- **e** 47.5 ± 0.5 mV (*1 mark*)
- **f** 366 000 ± 1000 J (*1 mark*)

4 Calculate the absolute uncertainty in these measurements.

- **a** 1200 W ± 10% (*1 mark*)
- **b** 34.1 m ± 1% (*1 mark*)
- **c** 330 000 Ω ± 0.5% (*1 mark*)
- **d** 0.008 00 m ± 1% (*1 mark*)

5 Calculate the absolute and percentage uncertainty in the total mass of suitcases of masses x, y, and z.

$x = 23.3\ (\pm\ 0.1)$ kg, $y = 18\ (\pm\ 1)$ kg, $z = 14.7\ (\pm\ 0.5)$ kg (*2 marks*)

Combining uncertainties

Introduction

In a calculation, if several of the quantities have uncertainties then these will all contribute to the uncertainty in the answer. The following rules will help you calculate the uncertainty in your final answers.

- When quantities are added, the uncertainty is the sum of the *absolute* uncertainties.
- When quantities are subtracted, the uncertainty is also the sum of the *absolute* uncertainties.
- When quantities are multiplied, the *total percentage* uncertainty is the sum of the *percentage* uncertainties.
- When quantities are divided, the *total percentage* uncertainty is also the sum of the *percentage* uncertainties.
- When a quantity is raised to the power n, the *total percentage* uncertainty is n multiplied by the *percentage* uncertainty – for example, for a quantity x^2, total percentage uncertainty $= 2 \times$ percentage uncertainty in x.

Worked example

Question

A current of 2.8 (± 0.1) A passes through a kettle element. The mains power supply is 230 (± 12) V.

Calculate the power transferred, including its uncertainty.

Answer

Step 1

Calculate the power.

$P = IV$

$P = (2.8\text{ A}) \times (230\text{ V}) = 644\text{ W}$

Step 2

Calculate the percentage uncertainties.

The percentage uncertainty in current $= \dfrac{0.1}{2.8} \times 100\% = 3.57\%$

The percentage uncertainty in voltage $= \dfrac{12}{230} \times 100\% = 5.22\%$

The percentage uncertainty in power $= 3.57\% + 5.22\% = 8.79\% = 9\%$ (to nearest %)

Step 3

Calculate the absolute uncertainty in the power.

The absolute uncertainty $= \dfrac{9}{100} \times 644\text{ W} = 58.0\text{ W}$

Step 4

State the answer with units.

Power = 644 (± 58) W

Questions

6 A piece of string 1.000 (± 0.002) m is cut from a ball of string of length 100.000 (± 0.002) m. Calculate the length of the remaining string and the uncertainty in this length. (*2 marks*)

7 A runner completes 100 (± 0.02) m in 18.6 (± 0.2) s. Calculate his average speed and the uncertainty in this value. (*2 marks*)

8 A car accelerates, with constant acceleration, from 24 (± 1) m s^{-1} to 31(± 2) m s^{-1} in 9.5 (± 0.1) s. Calculate the acceleration. State your answer with its absolute uncertainty. (*3 marks*)

9 A cube has a mass of 7.870 (± 0.001) kg and sides of length 10.0 (± 0.1) cm. Give the value of the density of the cube. (*2 marks*)

10 In a Young's slits experiment, two slits that are very close together are illuminated, and on a distant screen an interference pattern of light and dark fringes is seen. The separation of the fringes can be used to calculate the wavelength of the light. In a demonstration of this experiment:

- the double slit separation, a = 0.20 (± 0.01) mm
- the distance from the slits to the screen, D = 4.07 (± 0.01) m
- the distance between two adjacent bright fringes x = 12.0 (± 0.05) mm.

The equation for calculating wavelength is $\lambda = \dfrac{ax}{D}$.

a Calculate:

i the wavelength, λ, of the light (*1 mark*)

ii the absolute uncertainty in the wavelength. (*2 marks*)

b The distance between 11 fringes (10 spaces) = 120.0 (± 0.05) mm. Using this value, calculate the new absolute uncertainty in the wavelength. (*2 marks*)

c Comment on whether the uncertainty in the wavelength could be significantly reduced by increasing the number of fringes measured to, for example, 20 or more. (*1 mark*)

Maths skills links to other areas

You may also need to calculate uncertainties when considering precision and accuracy of measurements and data, including margins of error, percentage errors, and uncertainties in apparatus.

Scalars and vectors

Specification references

- *2.3.1 a) b) c)*
- *M0.6*
- *M4.1, M4.2, M4.4, M4.5*

Introduction

You will have met vector and scalar quantities in GCSE Maths and Physics so should know that examples of **scalars**, which have **only magnitude**, are speed, distance, mass, energy, and time. They are always positive.

Vector quantities have **magnitude and direction**, and examples are velocity (speed in a certain direction), displacement (distance travelled in a certain direction), force, acceleration, and momentum. All of these quantities can be negative or positive (the negative sign indicating the opposite direction).

Learning outcomes

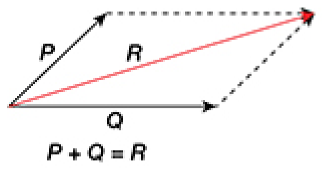
After completing the worksheet you should be able to:

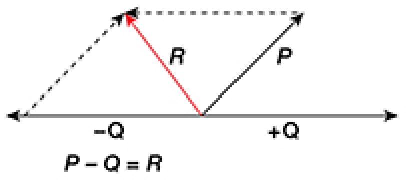
- define and identify scalar and vector quantities
- carry out vector addition and subtraction
- use a vector triangle to determine the resultant of any two coplanar vectors.

Background

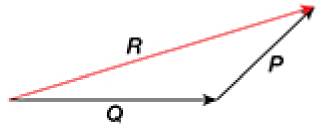
To combine any two vectors you can use any of the following methods.

11 You can draw a scale diagram where the lengths of the sides of a parallelogram represent the magnitude of the vectors. The diagonal shows us the magnitude and direction of the resultant force.





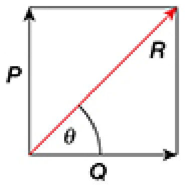
12 You can draw a scale diagram of a triangle, and represent the vectors in magnitude and direction by the three sides of a closed triangle taken in order.



13 If vectors are at right angles to each other you can use Pythagoras' theorem.

$\boldsymbol{P}^2 + \boldsymbol{Q}^2 = \boldsymbol{R}^2$

You can then use $\tan\theta = \dfrac{P}{Q}$ to find θ and define the angle



If using Method 1 you can follow these steps:

- choose a scale
- draw a horizontal line to represent one vector
- draw a line from the end of the first vector at the correct angle to represent the second vector
- join the beginning of the first vector to the end of second. This will be the resultant vector.

Method 3 is quicker and can be more accurate (depending on your drawing skills) than Methods 1 and 2.

Worked example

Question

Find the resultant of the forces shown in Figure 1.

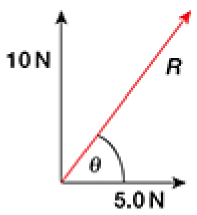


Figure 1

Answer

Step 1

The forces are at right angles so we can use Pythagoras' theorem.
Don't forget to take the square root in the calculation.

$R^2 = 5^2 + 10^2$

$R^2 = 25 + 100$

$R^2 = 125$

$R = \sqrt{125}$

$R = 11\ \text{N}$

Step 2

Calculate the angle, θ, using the formula for its sin, cos, or tan.
It may be best to use the formula for its tan since it uses the vector values you've already been given but assuming that the resultant has been calculated correctly, any of the three can be used.

$$\tan\theta = \frac{10}{5}$$

$$\tan\theta = 2$$

$$\theta = 63°$$

Questions

14 Give four examples of vector quantities and four examples of scalar quantities, including their units. (*4 marks*)

15 A car's engines generate a forward driving force of 52.7 N against which an opposite, frictional force of 36.5 N is acting. Calculate the magnitude and direction of the resultant force on the car. (*1 mark*)

16 Find the magnitude and direction of the resultant force formed by a 3.0 N and a 4.0 N force acting at right angles to each other using

a Pythagoras' theorem

b a triangle of forces (*3 marks*)

17 A river is flowing at 9.5 m s^{-1}. A child tries to swim across at 3.0 m s^{-1} at right angles to the direction of the flow of the river. Calculate the magnitude and direction of their resultant velocity. (*2 marks*)

18 An aircraft flies south at a velocity of 55 m s^{-1}, the wind is blowing from the west at a constant velocity of 15 m s^{-1}. Calculate the magnitude and direction of the resultant velocity of the aircraft. (*2 marks*)

19 Find the magnitude and direction of the resultant force formed by a horizontal force of 2.0 N, and a 10 N force at an angle of 60° to the horizontal using a scale diagram. *You will need a protractor to do this.* (*2 marks*)

Using scalars and vectors

Specification references

- *2.3.1 a) b) c) d)*
- *M0.6 Use calculators to handle sin x, cos x, and tan x when x is expressed in degrees or radians*
- *M4.2 Visualise and represent 2D and 3D forms*
- *M4.4 Use Pythagoras' theorem and the angle sum of a triangle*
- *M4.5 Use sin, cos, and tan in physical problems*

Learning outcomes

After completing the worksheet you should be able to:

- show and apply knowledge and understanding of scalar and vector quantities
- solve problems involving vector addition and subtraction
- use a vector triangle to determine the resultant of any two coplanar vectors.

Introduction

Scalar quantities have magnitude but no direction. For example, speed, distance, and time are all scalar quantities.

Vector quantities have magnitude and direction. Velocity is a vector quantity: it is speed in a certain direction. When we calculate velocity, v, we need to know the displacement, s. Displacement is also a vector: it is the distance travelled in a certain direction. The direction of a vector is sometimes indicated by giving an angle to a reference direction, for example, north. Sometimes a vector has a positive direction and a negative direction, in this case, the negative direction is opposite to the positive direction.

Acceleration is the rate of change of velocity, not of speed. This means that it is a vector. Consider an object moving in a circle at constant speed. Its direction is constantly changing, which means its velocity is changing. Therefore, it is accelerating.

When you add or subtract vectors, you must take the direction into account. Figure 1 shows that walking 3 m from **A** to **B**, and then turning through 30° and walking 2 m to **C**, has the same effect as walking directly from **A** to **C**. $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are vectors and are shown by a single arrowhead on a vector diagram. $\overrightarrow{AC}$ is the resultant vector, shown by the double arrowhead.

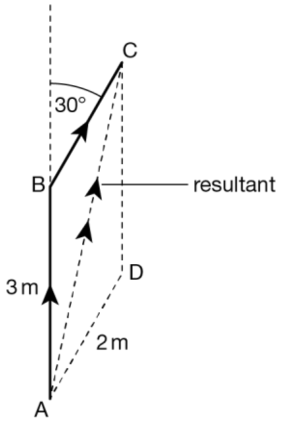
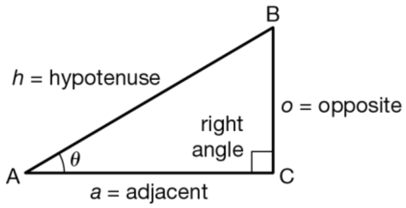


Figure 1

To combine any two vectors, we can draw a triangle like **ABC** in Figure 1, where the lengths of the sides represent the magnitude of the vector (for example, forces of 30 N and 20 N). The third side of the triangle shows us the magnitude and direction of the resultant force. Careful drawing of a scale diagram allows us to measure these.

Notice that if the vectors are combined by drawing them in the opposite order (**AD** and **DC** in Figure 1) these are the other two sides of a parallelogram and give the same resultant. If you draw both vectors so they start from point **A**, their resultant will be the diagonal of the parallelogram.

In solving problems with triangles, remember the angles in a triangle add up to 180°. In a right-angled triangle this means the other two angles add up to 90°.



For a right-angled triangle as shown, Pythagoras' theorem says:

$h^2 = o^2 + a^2$

Also:

$\sin\theta = \dfrac{o}{h}$, $\cos\theta = \dfrac{a}{h}$, $\tan\theta = \dfrac{o}{a}$ (some people remember: soh cah toa)

Worked example

Question

A sub-atomic particle experiences two forces at right angles, one of 2.0×10^{-15} N the other 3.0×10^{-15} N. Calculate the resultant force on the particle.

Answer

Step 1

Draw a diagram showing the two forces on the particle.

You can either draw the two forces acting on the particle at the same point, with the resultant as the diagonal of the rectangle formed, or consider the forces acting one after the other with the resultant as the third side of the triangle. Figure 2 shows both of these diagrams. In each the resultant is represented by *F*.

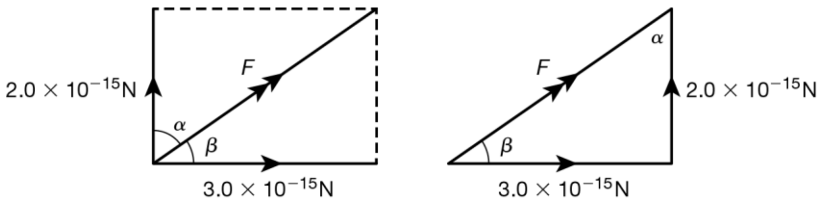


Figure 2

Step 2

Use Pythagoras' theorem to calculate the magnitude of the resultant, *F*.

$F^2 = (2.0 \times 10^{-15}\ \mathrm{N})^2 + (3.0 \times 10^{-15}\ \mathrm{N})^2$

$\quad = (4.0 \times 10^{-30} + 9.0 \times 10^{-30})\ \mathrm{N}^2$

Step 3

Don't forget to take the square root of your answer.

$F = \sqrt{13\ ´\ 10^{-30}}\ \mathrm{N}$

Step 4

Write your answer to the same number of significant figures as the question and with the correct units.

$F = 3.6 \times 10^{-15}\ \mathrm{N}$

Step 5

Either calculate the angle to the vertical using tan α, or calculate the angle to the horizontal using tan β.

$$\tan \alpha = \frac{3.0\ ´\ 10^{-15}\ \mathrm{N}}{2.0\ ´\ 10^{-15}\ \mathrm{N}} = 1.5$$

Use your calculator to find α.

$\alpha = \tan^{-1} 1.5 = 56°$

Or:

$$\tan \beta = \frac{2.0\ ´\ 10^{-15}\ \mathrm{N}}{3.0\ ´\ 10^{-15}\ \mathrm{N}} = 0.67$$

Use your calculator to find β.

$\beta = \tan^{-1} 0.67 = 34°$

Step 6

Don't forget to write out your final answer.

If you used angle α:

Resultant force = 3.6×10^{-15} N at 56° to the 2.0×10^{-15} N force

Or, if you used angle β:

Resultant force = 3.6×10^{-15} N at 34° to the 3.0×10^{-15} N force

Questions

20 Divide these quantities into vectors and scalars. (*3 marks*)

density
electric charge
electrical resistance
energy
field strength
force
friction
frequency
mass
momentum
power
voltage
volume
weight
work done

21 Divide these data into vectors and scalars. (*2 marks*)

$3\ \text{m s}^{-1}$

$+20\ \text{m s}^{-1}$

100 m NE

50 km

–5 cm

10 km S30°W

22 a Sketch a vector triangle showing a force of 3.0 N and a force of 4.0 N acting at right angles, and the resultant of the two vectors. (*2 marks*)

b Determine the magnitude and direction of the resultant force. (*2 marks*)

23 Find the resultant force of a 5.0 N and 12.0 N force acting at right angles. (*2 marks*)

24 A ship is cruising at 9.4 m s^{-1} and a boy runs across the deck, at right angles to the direction of the ship, at 3.0 m s^{-1}. Calculate his resultant velocity. (*2 marks*)

25 An aircraft flies east at a speed of 53 m s^{-1}. The wind is blowing from the north at a constant 16 m s^{-1}. What is the resultant velocity of the aircraft? (*2 marks*)

26 Two tugboats are towing a ship in a straight line. Tug A is pulling with a force of 50 kN at 60° to the direction in which the ship is moving. Tug B is pulling at 30° to the direction in which the ship is moving. Draw a sketch and then calculate the magnitude of:

a the resultant force on the ship (*1 mark*)

b the force from tug B. (*1 mark*)

Maths skills links to other areas

You may need to do similar calculations and draw vector diagrams in Topic 3.8 *Projectile motion*, and Topic 4.7 *Triangle of forces*.

Resolving vectors

Specification references

- *2.3.1 a) b) c) d)*
- *M0.6*
- *M4.1, M4.2, M4.4, M4.5*

Introduction

In GCSE Maths and Physics you will have become familiar with vector and scalar quantities, the difference being that scalars have only magnitude while vectors have both magnitude and direction. As a result, scalar quantities are always positive while vector quantities can be either positive or negative.

In some cases there may be several forces acting on the same object at different angles or at different points on the same object, and the resultant force vector cannot be found directly. In these cases one or more of the force vectors can be resolved into two components each before the resultant force vector can be calculated.

Learning outcomes

After completing the worksheet you should be able to:

- carry out vector addition and subtraction
- use a vector triangle to determine the resultant of any two coplanar vectors
- resolve a vector into two perpendicular components: $F_x = F\cos\theta$; $F_y = F\sin\theta$.

Background

To resolve a vector (force *F* in Figure 1) into two components you first split it into two vectors at right angles to each other, as shown in Figure 1. Note that these components *replace* the original force.

If the angle θ is between the original force *F* and the horizontal as shown in Figure 1:

- the horizontal component is $F\cos\theta$
- the vertical component is $F\sin\theta$.

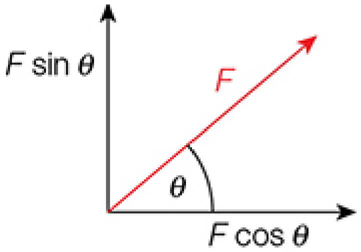


Figure 1

There are two main reasons why you may need to resolve forces.

27 To find the resultant of several forces acting at different angles.

To do this you will need to:

- draw a diagram showing the horizontal and vertical components of all the forces
- find the resultant forces acting at right angles to each other by combining the components in those directions
- combine the resultant perpendicular components using Pythagoras' theorem
- find the angle θ between the resultant force and *either* the vertical *or* horizontal to define its direction
- remember it is a **vector** and so must have *both* size and direction.

28 To find a force that balances other forces in order to show that an object is in equilibrium.

For example, finding the weight of a picture hung on a wall, given the tension in the string and the angle of the string to the horizontal (as in Figure 2).

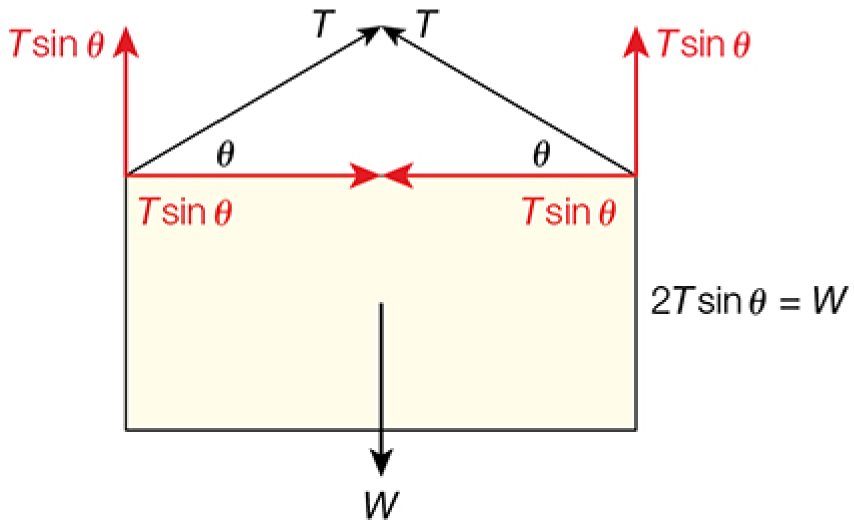


Figure 2

Worked example 1

Question

A picture is hanging on the wall, supported by two strings attached to its top corners. The tension in the strings is 3.0 N and the angle of the strings to the horizontal is 40°. Given this information, calculate the weight of the picture.

Answer

Step 1

Mark the force components on the diagram as shown in Figure 3.

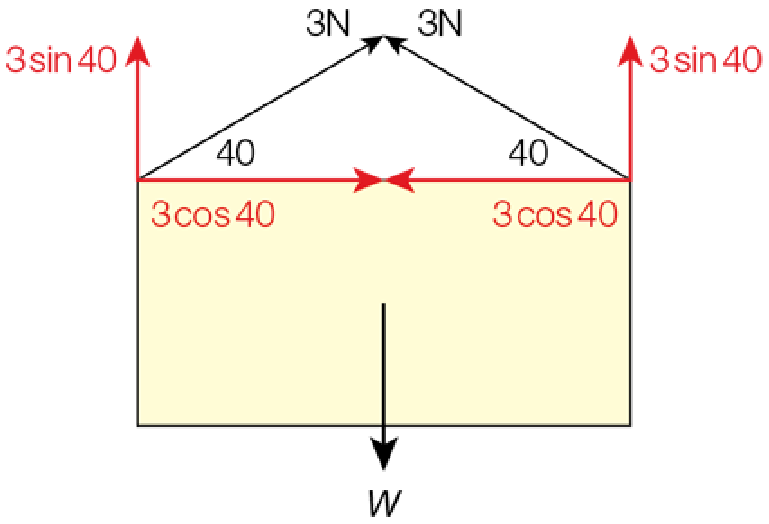


Figure 3

Step 2

Since the picture is in equilibrium, the total upward forces must be equal to the total downward forces.

The weight, *W*, of the picture must then be equal to the sum of the vertical components of the tension in the strings.

$W = 2 \times (3 \sin 40)$

$W = 3.9\ \text{N}$

Worked example 2

Question

Calculate the magnitude and direction of the resultant of all the forces shown in Figure 4.

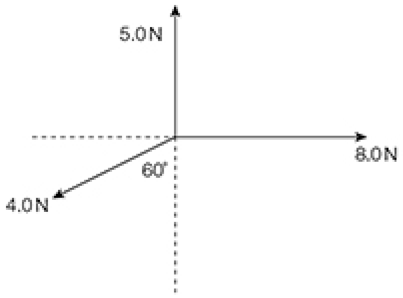


Figure 4

Answer

Step 1

First resolve the 4.0 N force into two components:

- 4.0 cos 60 in the vertical direction
- 4.0 sin 60 in the horizontal direction

The 5.0 N and 8.0 N forces are at right angles to each other so their resultant can be calculated using Pythagoras' theorem. In other words, there is no need to resolve these forces. If in doubt on which force(s) to resolve, look for those that are *not* at right angles to each other.

Step 2

Now mark the force components on the diagram as shown in Figure 5.

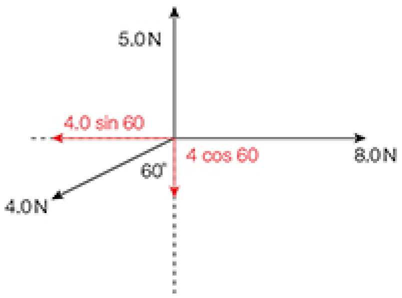


Figure 5

Step 3

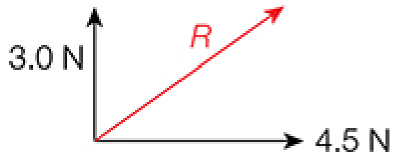
Find the resultant force in the horizontal and vertical directions. (Don't worry if you get a negative value, it simply means that the force is in the opposite direction.)

horizontal resultant force = 8 – (4 sin 60) = 4.5 N

vertical resultant force = 5 – (4 cos 60) = 3.0 N

Step 4

Redraw the diagram using the new vector values.



Step 5

Use Pythagoras' theorem to find the resultant force.

$R^2 = 3.0^2 + 4.5^2$

$R = \sqrt{29.5}$

$R = 5.4$ N

Step 6

Find the angle. If θ is the angle between R and the horizontal:

$\tan\theta = \dfrac{3.0}{4.5}$

$\tan\theta = 0.66$

$\theta = 34°$

Resultant force is 5.4 N at 34° to the horizontal.

Questions

29 Figure 6 shows a weight of 200 N hanging from the midpoint of a wire. Calculate the tension, ***T***, in the wire. (*2 marks*)

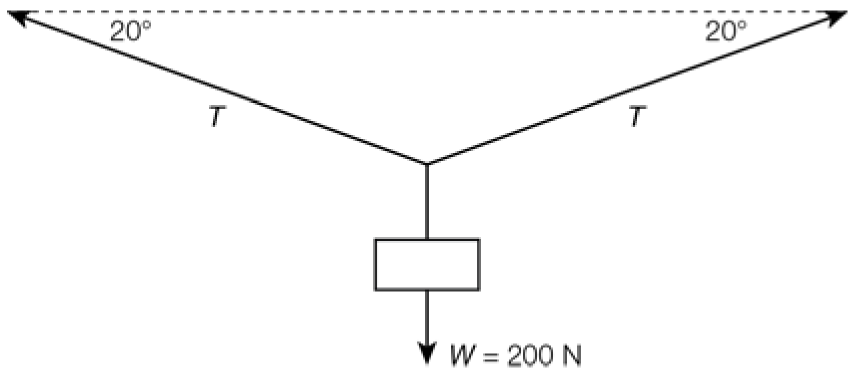
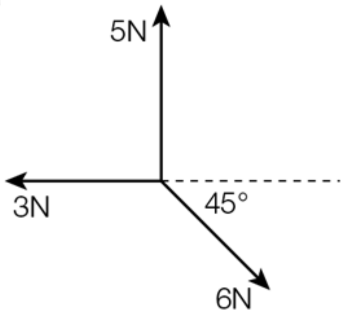


Figure 6

30 Find the magnitude and direction of the resultant of the forces shown in Figure 7. (*4 marks*)

a



b

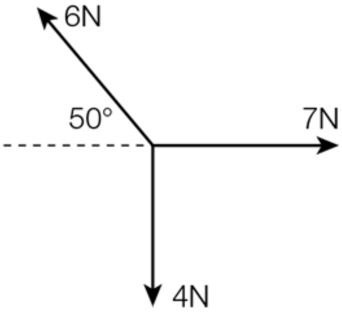


Figure 7

31 Two tug boats X and Y are towing a ship in a straight line into a harbour. Tug boat X is pulling with a force of 40.0 kN at 40° to the direction the ship is moving. Tug boat Y is pulling with a force of 51.5 kN at 30° to the direction the ship is moving.

Calculate the magnitude of the resultant, forward force on the ship. (*3 marks*)

Coplanar forces

Specification references

- *2.3.1 a) b) c) d)*

Learning outcomes

After completing the worksheet you should be able to:

- state the definitions of vectors and scalars
- resolve a force into its vertical and horizontal components
- combine vectors by calculation and by scale drawing.

Questions

32 Define a vector quantity. (*1 mark*)

33 Identify which of the following quantities are vectors and which are scalars. (*4 marks*)

weight distance momentum kinetic energy mass
velocity displacement acceleration temperature upthrust

34 A rope is pulled with a tension of 50 N at an angle of 30° to the horizontal.

a Calculate the horizontal component of the tension. (*1 mark*)

b Calculate the vertical component of the tension. (*1 mark*)

35 A parent is pushing their child on a swing. They stop the child momentarily by pulling on the swing seat with a horizontal force. The swing makes an angle of 40° to the vertical. The child and swing seat have a mass of 25 kg.

a Draw a free-body diagram of the forces acting on the child. (*3 marks*)

b Determine the tension in the swing. (*2 marks*)

c Find the force, *F*, with which the parent is holding the swing. (*2 marks*)

36 *From OCR Physics A G481 Mechanics May 2011 (Question 3c)*

Figure 1 shows the forces acting on a stage light of weight 120 N held stationary by two separate cables.

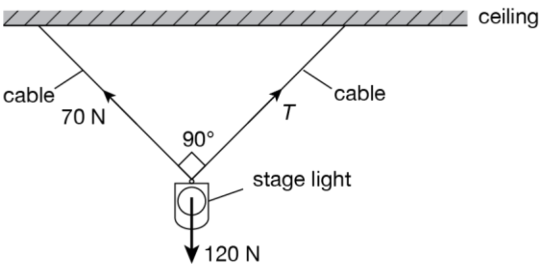


Figure 1

The angle between the two cables is 90°. One cable has tension 70 N and the other has tension *T.*

a State the magnitude and the direction of the **resultant** of the tensions in the two cables. (*2 marks*)

b Sketch a labelled vector triangle for the forces acting on the stage light. Hence, determine the magnitude of the tension *T.* (*4 marks*)