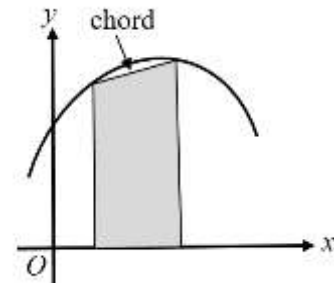# Area under a graph

> **A LEVEL LINKS**
> **Scheme of work:** 7b. Definite integrals and areas under curves

Key points

- To estimate the area under a curve, draw a chord between the two points you are finding the area between and straight lines down to the horizontal axis to create a trapezium. The area of the trapezium is an approximation for the area under a curve.
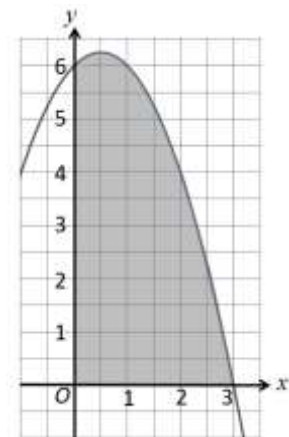
- The area of a trapezium = $\frac{1}{2}h(a+b)$
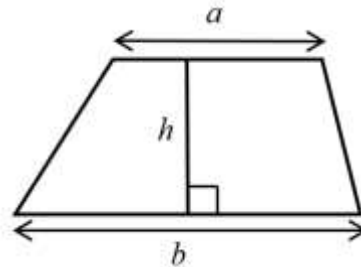
Examples

Example 1   Estimate the area of the region between the curve $y = (3 - x)(2 + x)$ and the x-axis from $x = 0$ to $x = 3$. Use three strips of width 1 unit.

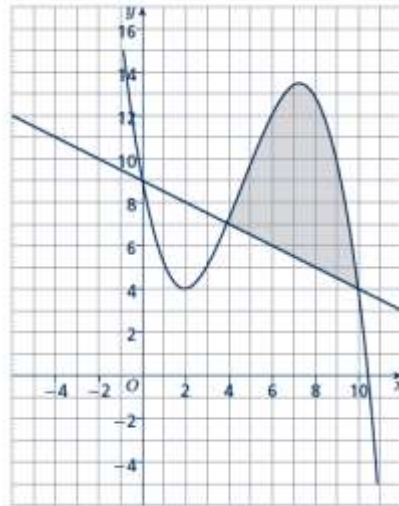| x	0	1	2	3
$y = (3 - x)(2 + x)$	6	6	4	0

Trapezium 1:
$a_1 = 6 - 0 = 6$, $b_1 = 6 - 0 = 6$
Trapezium 2:
$a_2 = 6 - 0 = 6$, $b_2 = 4 - 0 = 4$
Trapezium 3:
$a_3 = 4 - 0 = 4$, $a_3 = 0 - 0 = 0$

1   Use a table to record the value of y on the curve for each value of x.

2   Work out the dimensions of each trapezium. The distances between the y-values on the curve and the x-axis give the values for a.

$\dfrac{1}{2}h(a_1+b_1)=\dfrac{1}{2}\times 1(6+6)=6$ $\dfrac{1}{2}h(a_2+b_2)=\dfrac{1}{2}\times 1(6+4)=5$ $\dfrac{1}{2}h(a_3+b_3)=\dfrac{1}{2}\times 1(4+0)=2$	**3**	Work out the area of each trapezium. $h=1$ since the width of each trapezium is 1 unit.
Area $=6+5+2=13$ units^2	**4**	Work out the total area. Remember to give units with your answer.

Example 2 Estimate the shaded area.
Use three strips of width 2 units.

x	4	6	8	10
y	7	12	13	4

1 Use a table to record y on the curve for each value of x.

x	4	6	8	10
y	7	6	5	4

2 Use a table to record y on the straight line for each value of x.

Trapezium 1:
$a_1=7-7=0$, $b_1=12-6=6$
Trapezium 2:
$a_2=12-6=6$, $b_2=13-5=8$
Trapezium 3:
$a_3=13-5=8$, $a_3=4-4=0$

3 Work out the dimensions of each trapezium. The distances between the y-values on the curve and the y-values on the straight line give the values for a.

$\dfrac{1}{2}h(a_1+b_1)=\dfrac{1}{2}\times 2(0+6)=6$

$\dfrac{1}{2}h(a_2+b_2)=\dfrac{1}{2}\times 2(6+8)=14$

$\dfrac{1}{2}h(a_3+b_3)=\dfrac{1}{2}\times 2(8+0)=8$

4 Work out the area of each trapezium. $h=2$ since the width of each trapezium is 2 units.

Area $=6+14+8=28$ units^2

5 Work out the total area. Remember to give units with your answer.
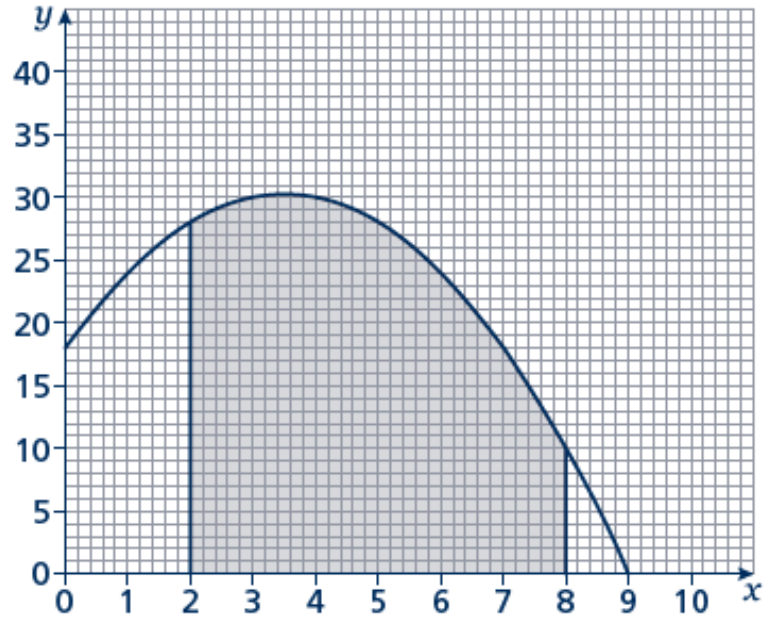
Practice

1 Estimate the area of the region between the curve $y = (5 - x)(x + 2)$ and the x-axis from $x = 1$ to $x = 5$.
Use four strips of width 1 unit.

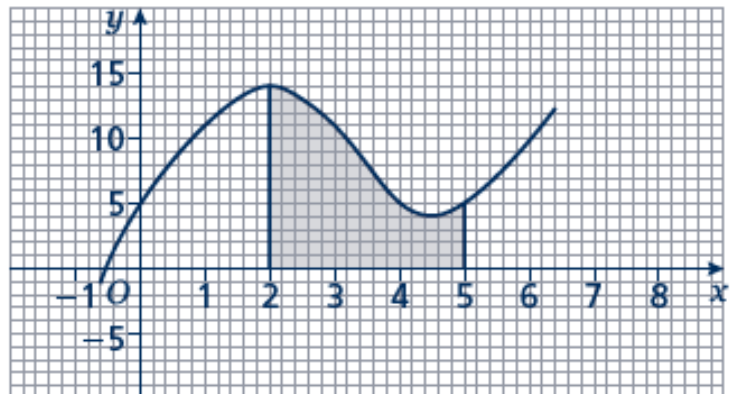**2** Estimate the shaded area shown on the axes.
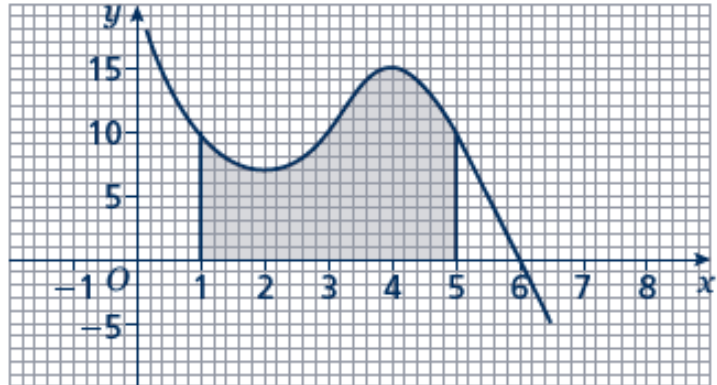Use six strips of width 1 unit.

3 Estimate the area of the region between the curve $y = x^2 - 8x + 18$ and the x-axis from $x = 2$ to $x = 6$.
Use four strips of width 1 unit.

4 Estimate the shaded area.
Use six strips of width $\frac{1}{2}$ unit.

5 Estimate the area of the region between the curve $y = -x^2 - 4x + 5$ and the
x-axis from $x = -5$ to $x = 1$.
Use six strips of width 1 unit.

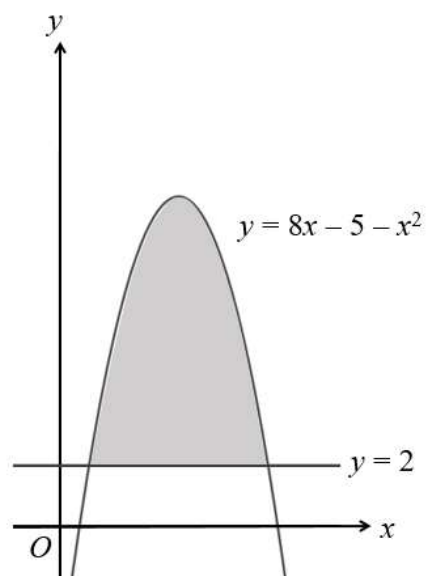**6** Estimate the shaded area.
Use four strips of equal width.

7 Estimate the area of the region between the curve $y = -x^2 + 2x + 15$ and the
x-axis from $x = 2$ to $x = 5$.
Use six strips of equal width.

8 Estimate the shaded area.
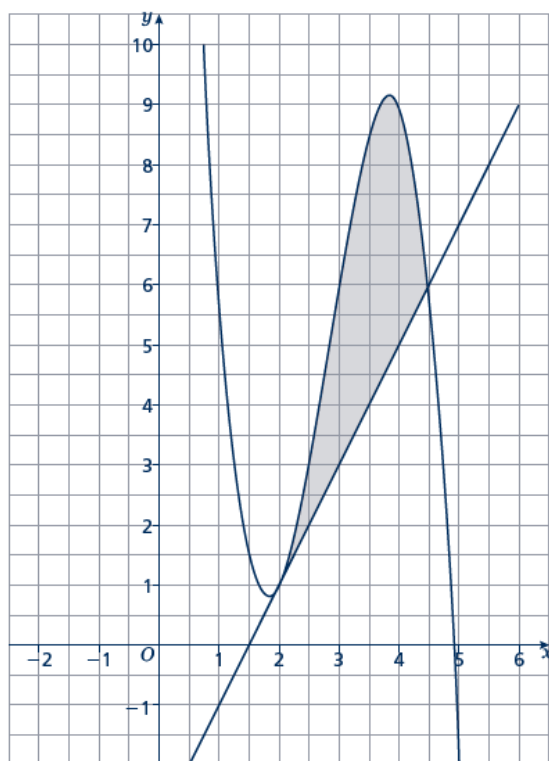Use seven strips of equal width.
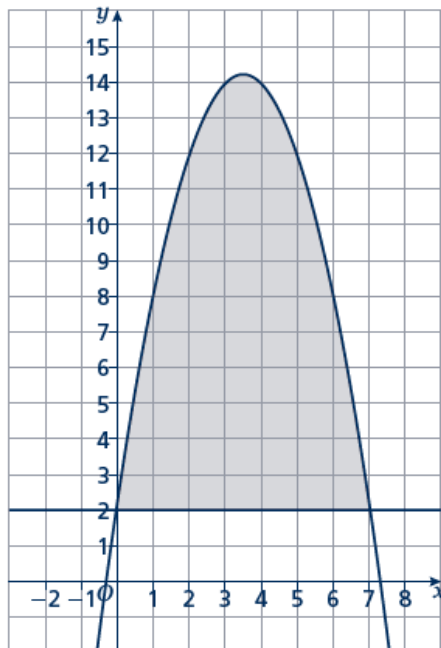
Extend

9 The curve $y = 8x - 5 - x^2$ and the line $y = 2$
are shown in the sketch.
Estimate the shaded area using six strips
of equal width.

10 Estimate the shaded area using five
strips of equal width.

Answers

1   34 units^2

2   149 units^2

3   14 units^2

4 $25\frac{1}{4}$ units^2

5   35 units^2

6   42 units^2

7 $26\frac{7}{8}$ units^2

8   56 units^2

9   35 units^2

10 $6\frac{1}{4}$ units^2