

Quadratic inequalities

A LEVEL LINKS

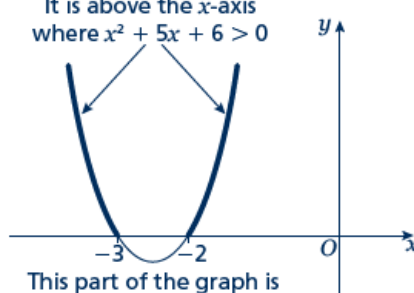
Scheme of work: 1d. Inequalities – linear and quadratic (including graphical solutions)

Key points

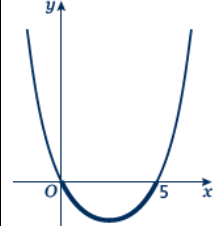
- First replace the inequality sign by = and solve the quadratic equation.
- Sketch the graph of the quadratic function.
- Use the graph to find the values which satisfy the quadratic inequality.

Examples

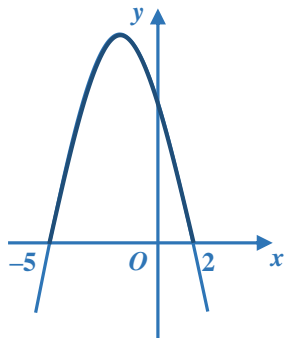
Example 1 Find the set of values of x which satisfy $x^2 + 5x + 6 > 0$

<p> $x^2 + 5x + 6 = 0$ $(x + 3)(x + 2) = 0$ $x = -3$ or $x = -2$ </p> <p> It is above the x-axis where $x^2 + 5x + 6 > 0$ </p>  <p> This part of the graph is not needed as this is where $x^2 + 5x + 6 < 0$ </p> <p> $x < -3$ or $x > -2$ </p>	<ol style="list-style-type: none"> 1 Solve the quadratic equation by factorising. 2 Sketch the graph of $y = (x + 3)(x + 2)$ 3 Identify on the graph where $x^2 + 5x + 6 > 0$, i.e. where $y > 0$ 4 Write down the values which satisfy the inequality $x^2 + 5x + 6 > 0$
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Example 2 Find the set of values of x which satisfy $x^2 - 5x \leq 0$

<p> $x^2 - 5x = 0$ $x(x - 5) = 0$ $x = 0$ or $x = 5$ </p>  <p> $0 \leq x \leq 5$ </p>	<ol style="list-style-type: none"> 1 Solve the quadratic equation by factorising. 2 Sketch the graph of $y = x(x - 5)$ 3 Identify on the graph where $x^2 - 5x \leq 0$, i.e. where $y \leq 0$ 4 Write down the values which satisfy the inequality $x^2 - 5x \leq 0$
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Example 3 Find the set of values of x which satisfy $-x^2 - 3x + 10 \geq 0$

$-x^2 - 3x + 10 = 0$ $(-x + 2)(x + 5) = 0$ $x = 2 \text{ or } x = -5$  $-5 \leq x \leq 2$	<ol style="list-style-type: none"> 1 Solve the quadratic equation by factorising. 2 Sketch the graph of $y = (-x + 2)(x + 5) = 0$ 3 Identify on the graph where $-x^2 - 3x + 10 \geq 0$, i.e. where $y \geq 0$ 3 Write down the values which satisfy the inequality $-x^2 - 3x + 10 \geq 0$
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Practice

- 1 Find the set of values of x for which $(x + 7)(x - 4) \leq 0$
- 2 Find the set of values of x for which $x^2 - 4x - 12 \geq 0$
- 3 Find the set of values of x for which $2x^2 - 7x + 3 < 0$
- 4 Find the set of values of x for which $4x^2 + 4x - 3 > 0$
- 5 Find the set of values of x for which $12 + x - x^2 \geq 0$

Extend

Find the set of values which satisfy the following inequalities.

- 6 $x^2 + x \leq 6$
- 7 $x(2x - 9) < -10$
- 8 $6x^2 \geq 15 + x$

Answers

1 $-7 \leq x \leq 4$

2 $x \leq -2$ or $x \geq 6$

3 $\frac{1}{2} < x < 3$

4 $x < -\frac{3}{2}$ or $x > \frac{1}{2}$

5 $-3 \leq x \leq 4$

6 $-3 \leq x \leq 2$

7 $2 < x < 2\frac{1}{2}$

8 $x \leq -\frac{3}{2}$ or $x \geq \frac{5}{3}$