

Completing the square

A LEVEL LINKS

Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Completing the square for a quadratic rearranges $ax^2 + bx + c$ into the form $p(x + q)^2 + r$
- If $a \neq 1$, then factorise using a as a common factor.

Examples

Example 1 Complete the square for the quadratic expression $x^2 + 6x - 2$

$$\begin{aligned}x^2 + 6x - 2 \\= (x + 3)^2 - 9 - 2 \\= (x + 3)^2 - 11\end{aligned}$$

- 1 Write $x^2 + bx + c$ in the form $\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$
- 2 Simplify

Example 2 Write $2x^2 - 5x + 1$ in the form $p(x + q)^2 + r$

$$\begin{aligned}2x^2 - 5x + 1 \\= 2\left(x^2 - \frac{5}{2}x\right) + 1 \\= 2\left[\left(x - \frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2\right] + 1 \\= 2\left(x - \frac{5}{4}\right)^2 - \frac{25}{8} + 1 \\= 2\left(x - \frac{5}{4}\right)^2 - \frac{17}{8}\end{aligned}$$

- 1 Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2 + \frac{b}{a}x\right) + c$
- 2 Now complete the square by writing $x^2 - \frac{5}{2}x$ in the form $\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$
- 3 Expand the square brackets – don't forget to multiply $\left(\frac{5}{4}\right)^2$ by the factor of 2
- 4 Simplify

Practice

- 1** Write the following quadratic expressions in the form $(x + p)^2 + q$
a $x^2 + 4x + 3$ **b** $x^2 - 10x - 3$
c $x^2 - 8x$ **d** $x^2 + 6x$
e $x^2 - 2x + 7$ **f** $x^2 + 3x - 2$

- 2** Write the following quadratic expressions in the form $p(x + q)^2 + r$
a $2x^2 - 8x - 16$ **b** $4x^2 - 8x - 16$
c $3x^2 + 12x - 9$ **d** $2x^2 + 6x - 8$

- 3** Complete the square.
a $2x^2 + 3x + 6$ **b** $3x^2 - 2x$
c $5x^2 + 3x$ **d** $3x^2 + 5x + 3$

Extend

- 4** Write $(25x^2 + 30x + 12)$ in the form $(ax + b)^2 + c$.

Answers

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|----------|----------|---|----------|---|
| 1 | a | $(x + 2)^2 - 1$ | b | $(x - 5)^2 - 28$ |
| | c | $(x - 4)^2 - 16$ | d | $(x + 3)^2 - 9$ |
| | e | $(x - 1)^2 + 6$ | f | $\left(x + \frac{3}{2}\right)^2 - \frac{17}{4}$ |
| 2 | a | $2(x - 2)^2 - 24$ | b | $4(x - 1)^2 - 20$ |
| | c | $3(x + 2)^2 - 21$ | d | $2\left(x + \frac{3}{2}\right)^2 - \frac{25}{2}$ |
| 3 | a | $2\left(x + \frac{3}{4}\right)^2 + \frac{39}{8}$ | b | $3\left(x - \frac{1}{3}\right)^2 - \frac{1}{3}$ |
| | c | $5\left(x + \frac{3}{10}\right)^2 - \frac{9}{20}$ | d | $3\left(x + \frac{5}{6}\right)^2 + \frac{11}{12}$ |
| 4 | | $(5x + 3)^2 + 3$ | | |