# Surds and rationalising the denominator

**A LEVEL LINKS**
**Scheme of work:** 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

## Key points

- A surd is the square root of a number that is not a square number, for example $\sqrt{2}, \sqrt{3}, \sqrt{5},$ etc.
- Surds can be used to give the exact value for an answer.
- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
- To rationalise the denominator means to remove the surd from the denominator of a fraction.
- To rationalise $\frac{a}{\sqrt{b}}$ you multiply the numerator and denominator by the surd $\sqrt{b}$
- To rationalise $\frac{a}{b+\sqrt{c}}$ you multiply the numerator and denominator by $b-\sqrt{c}$

## Examples

**Example 1** Simplify $\sqrt{50}$

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|---|---|
| $\sqrt{50} = \sqrt{25 \times 2}$ | **1** Choose two numbers that are factors of 50. One of the factors must be a square number |
| $= \sqrt{25} \times \sqrt{2}$ | **2** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ |
| $= 5 \times \sqrt{2}$ | **3** Use $\sqrt{25} = 5$ |
| $= 5\sqrt{2}$ | |

**Example 2** Simplify $\sqrt{147} - 2\sqrt{12}$

| | |
|---|---|
| $\sqrt{147} - 2\sqrt{12}$<br>$= \sqrt{49 \times 3} - 2\sqrt{4 \times 3}$ | **1** Simplify $\sqrt{147}$ and $2\sqrt{12}$. Choose two numbers that are factors of 147 and two numbers that are factors of 12. One of each pair of factors must be a square number |
| $= \sqrt{49} \times \sqrt{3} - 2\sqrt{4} \times \sqrt{3}$ | **2** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ |
| $= 7 \times \sqrt{3} - 2 \times 2 \times \sqrt{3}$ | **3** Use $\sqrt{49} = 7$ and $\sqrt{4} = 2$ |
| $= 7\sqrt{3} - 4\sqrt{3}$ | **4** Collect like terms |
| $= 3\sqrt{3}$ | |

**Example 3** Simplify $\left(\sqrt{7}+\sqrt{2}\right)\left(\sqrt{7}-\sqrt{2}\right)$

| | |
|---|---|
| $\left(\sqrt{7}+\sqrt{2}\right)\left(\sqrt{7}-\sqrt{2}\right)$<br>$= \sqrt{49}-\sqrt{7}\sqrt{2}+\sqrt{2}\sqrt{7}-\sqrt{4}$<br>$= 7 - 2$<br>$= 5$ | **1** Expand the brackets. A common mistake here is to write $\left(\sqrt{7}\right)^2 = 49$<br>**2** Collect like terms:<br>$-\sqrt{7}\sqrt{2}+\sqrt{2}\sqrt{7}$<br>$= -\sqrt{7}\sqrt{2}+\sqrt{7}\sqrt{2} = 0$ |

**Example 4** Rationalise $\frac{1}{\sqrt{3}}$

| | |
|---|---|
| $\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$<br>$= \frac{1\times\sqrt{3}}{\sqrt{9}}$<br>$= \frac{\sqrt{3}}{3}$ | **1** Multiply the numerator and denominator by $\sqrt{3}$<br>**2** Use $\sqrt{9}=3$ |

**Example 5** Rationalise and simplify $\frac{\sqrt{2}}{\sqrt{12}}$

| | |
|---|---|
| $\frac{\sqrt{2}}{\sqrt{12}} = \frac{\sqrt{2}}{\sqrt{12}}\times\frac{\sqrt{12}}{\sqrt{12}}$<br>$= \frac{\sqrt{2}\times\sqrt{4\times3}}{12}$<br>$= \frac{2\sqrt{2}\sqrt{3}}{12}$<br>$= \frac{\sqrt{2}\sqrt{3}}{6}$ | **1** Multiply the numerator and denominator by $\sqrt{12}$<br>**2** Simplify $\sqrt{12}$ in the numerator. Choose two numbers that are factors of 12. One of the factors must be a square number<br>**3** Use the rule $\sqrt{ab}=\sqrt{a}\times\sqrt{b}$<br>**4** Use $\sqrt{4}=2$<br>**5** Simplify the fraction:<br>$\frac{2}{12}$ simplifies to $\frac{1}{6}$ |

**Example 6** Rationalise and simplify $\dfrac{3}{2+\sqrt{5}}$

| | |
|---|---|
| $\dfrac{3}{2+\sqrt{5}} = \dfrac{3}{2+\sqrt{5}} \times \dfrac{2-\sqrt{5}}{2-\sqrt{5}}$ | **1** Multiply the numerator and denominator by $2-\sqrt{5}$ |
| $= \dfrac{3\left(2-\sqrt{5}\right)}{\left(2+\sqrt{5}\right)\left(2-\sqrt{5}\right)}$ | **2** Expand the brackets |
| $= \dfrac{6-3\sqrt{5}}{4+2\sqrt{5}-2\sqrt{5}-5}$ | **3** Simplify the fraction |
| $= \dfrac{6-3\sqrt{5}}{-1}$ | **4** Divide the numerator by $-1$ Remember to change the sign of all terms when dividing by $-1$ |
| $= 3\sqrt{5}-6$ | |

## Practice

**1** Simplify.

**a** $\sqrt{45}$ **b** $\sqrt{125}$

**c** $\sqrt{48}$ **d** $\sqrt{175}$

**e** $\sqrt{300}$ **f** $\sqrt{28}$

**g** $\sqrt{72}$ **h** $\sqrt{162}$

**Hint**

One of the two numbers you choose at the start must be a square number.

**2** Simplify.

**a** $\sqrt{72}+\sqrt{162}$ **b** $\sqrt{45}-2\sqrt{5}$

**c** $\sqrt{50}-\sqrt{8}$ **d** $\sqrt{75}-\sqrt{48}$

**e** $2\sqrt{28}+\sqrt{28}$ **f** $2\sqrt{12}-\sqrt{12}+\sqrt{27}$

**Watch out!**

Check you have chosen the highest square number at the start.

**3** Expand and simplify.

**a** $(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})$ **b** $(3+\sqrt{3})(5-\sqrt{12})$

**c** $(4-\sqrt{5})(\sqrt{45}+2)$ **d** $(5+\sqrt{2})(6-\sqrt{8})$

**4** Rationalise and simplify, if possible.

**a** $\frac{1}{\sqrt{5}}$ **b** $\frac{1}{\sqrt{11}}$

**c** $\frac{2}{\sqrt{7}}$ **d** $\frac{2}{\sqrt{8}}$

**e** $\frac{2}{\sqrt{2}}$ **f** $\frac{5}{\sqrt{5}}$

**g** $\frac{\sqrt{8}}{\sqrt{24}}$ **h** $\frac{\sqrt{5}}{\sqrt{45}}$

**5** Rationalise and simplify.

**a** $\frac{1}{3-\sqrt{5}}$ **b** $\frac{2}{4+\sqrt{3}}$ **c** $\frac{6}{5-\sqrt{2}}$

## Extend

**6** Expand and simplify $\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right)$

**7** Rationalise and simplify, if possible.

**a** $\frac{1}{\sqrt{9}-\sqrt{8}}$ **b** $\frac{1}{\sqrt{x}-\sqrt{y}}$

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# Answers

**1** **a** $3\sqrt{5}$ **b** $5\sqrt{5}$

**c** $4\sqrt{3}$ **d** $5\sqrt{7}$

**e** $10\sqrt{3}$ **f** $2\sqrt{7}$

**g** $6\sqrt{2}$ **h** $9\sqrt{2}$

**2** **a** $15\sqrt{2}$ **b** $\sqrt{5}$

**c** $3\sqrt{2}$ **d** $\sqrt{3}$

**e** $6\sqrt{7}$ **f** $5\sqrt{3}$

**3** **a** $-1$ **b** $9-\sqrt{3}$

**c** $10\sqrt{5}-7$ **d** $26-4\sqrt{2}$

**4** **a** $\frac{\sqrt{5}}{5}$ **b** $\frac{\sqrt{11}}{11}$

**c** $\frac{2\sqrt{7}}{7}$ **d** $\frac{\sqrt{2}}{2}$

**e** $\sqrt{2}$ **f** $\sqrt{5}$

**g** $\frac{\sqrt{3}}{3}$ **h** $\frac{1}{3}$

**5** **a** $\frac{3+\sqrt{5}}{4}$ **b** $\frac{2(4-\sqrt{3})}{13}$ **c** $\frac{6(5+\sqrt{2})}{23}$

**6** $x-y$

**7** **a** $3+2\sqrt{2}$ **b** $\frac{\sqrt{x}+\sqrt{y}}{x-y}$

Pearson